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RGM Integro-differential equation for the scattering of two nuclei in the single channel approximation

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Abstract: A microscopic study of nucleus, nuclear scattering and reactions using Resonating Group Method (RGM) Bernheim et al. (1967) has been carried out with considerable success. In this method a nucleon - nucleon potential which is capable of explaining reasonably well the nucleon low energy scattering data Bouten et al. (1969) is used. Pauli's Exclusion Principle is also taken into account by employing fully antisymmetric wave functions. In this paper we have considered the simple case of elastic scattering of two clusters where all other channels and distortion effects are neglected.

Keywords: RGM • cluster • elastic scattering

Introduction

RGM was employed to study the problems of nuclear scattering and reactions in particular cases. Because of computational difficulties, only very light system could be investigated, which involve two s-shell nuclei in both the incident and outgoing channels. RGM is based on the consideration that because of the on the average attractive nature of nuclear forces there exist in nuclei relatively long-range co-relations which manifest themselves through the formation of nucleon clusters.

It was emphasized that the Pauli's exclusion principle causes these cluster co-relations to become strongest in the nuclear surface region. Such a cluster picture is referred to as the nuclear cluster model Vinciguerra et al. (1969). Resonating group method is a microscopic method which explicitly takes cluster co-relations into account. It uses entirely antisymmetric wavefunction that is why the Pauli's exclusion principle is fully taken into account. It also treats correctly total centre of mass motion.

Formulation of Single Channel approximation

Let us consider the elastic scattering of two nuclei A and B i.e., two clusters. In this approximation the projection equation Jansens et al. (1972) can be written as:

$$< \delta\psi_S \mid H - E_T \mid \psi_S > = 0 \text{ --- (1)}$$

Where, H is Hamiltonian for the total system and E_T is the total energy of the system. The trial wave function of the system can be written as:

$$\psi_S = A\,[\phi_A\,\phi_B\,\chi\,(\bar{R})\,\xi\,(\sigma,\tau)] \text{ --- (2)}$$

Where, A is antisymmetrization operator, ϕ_A and ϕ_B are the wave functions for the internal motion of nucleons in nuclei A and B respectively. χ (R) is relative motion of the two nuclei and is function of the distance $\bar{R}$ between their centre of mass, $\xi(\sigma,\tau)$ is spin-isospin function for the system. Equation (2) can be written in parameter $\bar{R}''$ coordinates but A does not depend on $\bar{R}''$. Now the projection equation for one channel approximation is:

$$< A\ [\phi_A(x_1)\,\phi_B(x_2)\,\xi(\sigma,\tau)\delta(R''-R)]\ |\,H-E_T|$$

$$A\ [\phi_A(x_1)\,\phi_B(x_2)\,\xi(\sigma,\tau)\,\chi(R)]> = 0 \quad \text{--- (3)}$$

Where x_1 and x_2 represent all the natural coordinates of nucleons in nuclei A and B. A is Hermitian operator, which commutes with the Hamiltonian operator and satisfies the relation:

$$A^2 = (N_A + N_B)\ !\ A \quad \text{--- (4)}$$

Where, N_A and N_B are the number of nucleons in the clusters A and B.

The Hamiltonian H can be arranged as:

$$H = H_A + H_B + H' \quad \text{--- (5)}$$

Where, H_A and H_B are the internal Hamiltonians of the clusters A and B. H' is the Hamiltonian of relative motion, given by:

$$H' = -\frac{h^2}{2\mu}\nabla_R^2 + V' \quad \text{--- (6)}$$

Where, μ is the reduced mass of the two clusters and

$$V' = \sum_{i\,\epsilon\,A}\sum_{j\,\epsilon\,B} V_{ij} \quad \text{--- (7)}$$

V_{ij} is the two nucleon potential. In general it contains all type of exchange operators.

The second term of equation (3) can be solved easily by replacing x $(\bar{R})$ by

$$\chi(\bar{R}) = \int \chi(\bar{R})\,\delta(\bar{R}-\bar{R}_1)\,dR' \quad \text{--- (8)}$$

Substituting equations (4), (5), (6), (7) and (8) in equation (3), the projection equation can be finally written as:

$$\left(-\frac{h^2}{2\mu}\nabla_R^2 + V_D(\bar{R}) - E\right) x(\bar{R}) + \int K(\bar{R},\bar{R}')\,\chi(\bar{R})\,d\bar{R}' = 0 \quad \text{--- (9)}$$

Where, $V_D(\bar{R})$ is the direct potential and $K(\bar{R},\bar{R}')$ is the non local potential Slight et al (1973).

The above equation is the integro-differential equation for the scattering in single channel approximation. This equation can be used to study the case of scattering of low energy nucleons from nucleus.

Discussion

RGM calculations are generally rather difficult to perform, especially when the number of nucleons, involved in the system is large. Indeed the requirement of antisymmetrization causes insurmountable computational difficulties and therefore the RGM cannot be expected to be useful in studying systems containing more than eight nucleons Saxena (1983). But with the development of Complex generator coordinate technique Sinha (1996), Sinha (2013) it becomes possible to perform scattering calculations for rather large systems such as scattering of ^{16}O and ^{40}Ca.

References

Bernheim M, Stoval T and Vinciguerra D (1967), Electron scattering from Be^9. *Nucl. Phys.* A97: 488-504.

Bouten M and Bouten MC (1969) Projected Hatree-Fock calculations for light nuclei:Normal parity states of Be^9, *Nucl. Phys.* A (127): 177-187.

Jansens JA, Peerdeman RT and Devcies C (1972) Nuclear charge radii of C^{12} and Be^9, Nucl. Phys., A (188): 337-352.

Saxena A (1983) High energy electron scattering with light nuclear systems, Ph.D. Thesis, Meerut University.

Sinha N (1996) Microscopic theoretical studies of light nuclear systems, Ph. D. Thesis, C.C.S. University.

Sinha P (2013) A normalized cluster model wave function of He^5 using CGCT, *Int. J. Theor. Appl. Sci.* 5(2):114-120.

Slight AG and Drake TE and Bishop GR (1973) Inelastic electron scattering and electromagnetic properties of Be^9. *Nucl. Phys*. A: 157-195.

Vinciguerra D and Stovall T (1969) Electron scattering from p-shell nuclei. *Nucl. Phys.* A (132): 410-432.

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